

Handout 4: Pascal's Wager

I. The Argument from Dominance

A. The Principle of Dominance

1. The Concept of Dominance

"The simplest special case occurs when one course of action is better no matter what the world is like"

- Ian Hacking, "The Logic of Pascal's Wager" (1972)

a. Decision Matrices

A *decision matrix* represents, for each act, A, that someone could perform at some time, and each state, S, that the world might be in, how good things would go for the agent if she were to do A, if the world is in S.

How good things would go for someone in some outcome is sometimes called the *utility* for this person of that outcome. We'll represent it with a number.

b. Examples of Decision Matrices

	<u>S1</u>	<u>S2</u>		<u>S1</u>	<u>S2</u>	<u>S3</u>
A1	100	-100	A1	100	100	300
A2	50	10	A2	50	100	300

c. Dominance Defined

An act *dominates* iff

- (i) in at least one state it brings about an outcome that is better for the agent than the outcome that would be brought about by any alternative act to it, and
- (ii) in no state does it bring about an outcome that is worse for the agent than the outcome that would be brought about by any alternative act to it.

In other words: an act dominates iff it might be better, and can't be worse.

In the decision matrix on the above right, act A1 dominates.

In the decision matrix on the above left, no act dominates.

2. The Principle

The Principle of Dominance: If an agent is in a situation in which one of her alternatives dominates, then she ought (prudentially speaking) to perform that alternative.

B. The Argument

Pascal: "Let us weigh the gain and the loss in wagering that God is. Let us estimate these two chances. If you gain, you gain all; if you lose, you lose nothing."

The Argument from Dominance

P1. The following decision matrix represents your situation with respect to the decision of whether to believe in God:

	<u>G</u>	<u>~G</u>
B	10,000	100
~B	-10,000	100

P2. If P1, then B dominates.

P3. If B dominates, then you ought to choose B.

C. Therefore, you ought to choose B (in other words, you should believe that God exists).

C. Potential Problems for the Argument from Dominance

1. Doxastic Voluntarism

Pascal: "Endeavor then to convince yourself, not by increase of proofs of God, but by the abatement of your passions. You would like to attain faith, and do not know the way; you would like to cure yourself of unbelief, and ask the remedy for it. Learn of those who have been bound like you, and who now stake all their possessions. These are people who know the way which you would follow, and who are cured of an ill of which you would be cured. Follow the way by which they began; by acting as if they believe, taking the holy water, having masses said, &c. Even this will naturally make you believe, and deaden your acuteness."

2. William James' Suspicion

Hacking: "many of us will share William James's suspicion that a person who becomes a believer for the reasons urged by Pascal is not going to get the pay-offs he hopes for" (189).

3. The Appeals of a Libertine Life:

Pascal: "'That is very fine. Yes, I must wager; but I may perhaps wager too much.'"

	<u>G</u>	<u>~G</u>
B	10,000	10
~B	-10,000	200

II. The Argument from Expected Value

A. The Principle of Expected Value

1. The Concept of Expected Value

"The *expected value*, or *expectation*, of [an act] is the average value of doing [it]"

- Ian Hacking, "The Logic of Pascal's Wager" (1972)

"In *decisions under risk*, the agent assigns subjective probabilities to the various states of the world. ... the *expected utility* ... of a given action can be calculated by a simple formula: for each state, multiply the utility that the action produces in that state by the state's probability; then, add these numbers."

- Alan Hájek, "Pascal's Wager," *The Stanford Encyclopedia of Philosophy*

Definition of 'expected value' (in English):

The *expected value* of an act, A, when there are several possible states (S1, S2, S3, etc.) that the world might be in, is the *sum* of these numbers:

- the probability that the world is in the first state (S1) × the utility or value for you of the world being in that state if you were to perform A;
- the probability that the world is in the second state (S2) × the utility of the world being in that state if you were to perform A;
- the probability that the world is in the third state (S3) × the utility of the world being in that state if you were to perform A;
- and so on, for all of the states that the world might be in.

Definition of 'expected value' (in Symbols):

$$EV(A) = [P(S1) \times V(A | S1)] + [P(S2) \times V(A | S2)] + [P(S3) \times V(A | S3)] + \dots$$

Probabilities are numbers between 0 and 1 representing how likely some claim is to be true; for example:

- '0' represents zero probability, or no chance of being true
- '1' represents 100% likelihood, or certainty of being true
- '.5' represent a 50/50 chance of being true
- '.25' represents a 1 in 4 chance of being true
- '.99' means highly likely
- '.01' means highly unlikely
- and so ...

'P(X)' stands for 'the probability that X is true'

'P(X | Y)' stands for 'the probability that X is true if Y is true'

2. Examples

- a. The Hike
- b. The Coin Bet
- c. The Libertine Life

3. The Principle of Expected Value

“In order to judge of what we ought to do in order to obtain a good and to avoid an evil, it is necessary to consider not only the good and evil in themselves, but also the probability of their happening and not happening”

- from *Port-Royal Logic* (1662), produced anonymously by Antoine Arnauld, Pierre Nicole, and possibly also Blaise Pascal

The Principle of Expected Value: One ought to maximize expected value (i.e., perform the act (or one of the acts) such that no alternative to it has a higher expected value than it).

B. The Argument

The Argument from Expected Value

P1. The following decision matrix represents your situation with respect to the decision of whether to believe in God:

	<u>G</u>	<u>~G</u>
<u>B</u>	10,000 ¹	10
<u>~B</u>	-10,000	200

P2. The following probability assignments represent how likely you think it is that God exists:

$$\begin{aligned} P(G) &= .5 \\ P(\sim G) &= .5 \end{aligned}$$

P3. If P1 and P2, then B maximizes expected value for you.

[*Try this exercise*: do all the calculations required to justify P3.]

P4. If B maximizes expected value for you, then you ought to choose B.

C. Therefore, you ought to choose B.

C. Problems for the Argument from Expected Value

1. Might $P(G) < .5$?

¹ Total faithfulness to the text would require that we use ‘∞’ here. For pedagogical reasons, we are electing to change only one column of the decision matrix at a time (the second column). We’ll consider the case of infinite utility in the third argument.

III. The Argument from Dominating Expected Value

A. The Principle of Dominating Expected Value

1. The Concept of Dominating Expected Value

An act has *dominating expected value* iff it maximizes expected value for all (non-zero) probability assignments.

In other words: an act has dominating expected value for you iff it maximizes your expected value no matter what you believe (with one rare exception).

2. The Principle of Dominating Expected Value

The Principle of Dominating Expected Value: If an agent is in a situation in which one of her alternatives has dominating expected value, then she ought to perform that alternative.

B. The Argument

The Argument from Dominating Expected Value

P1. The following decision matrix represents your situation with respect to the decision of whether to believe in God:

	<u>G</u>	<u>~G</u>
B	∞	10
~B	-10,000	200

Pascal: "But there is here an infinity of an infinitely happy life to gain, a chance of gain against a finite number of chances of loss, and what you stake is finite. It is all divided; wherever the infinite is and there is not an infinity of chances of loss against that of gain, there is no time to hesitate, you must give all."

Why $V(\sim B | G) \neq -\infty$:

Pascal: "justice to the outcast is less vast ... than mercy towards the elect."

P2. If P1, then B has dominating expected value.

P3. If B has dominating expected value, then you ought to choose B.

C. Therefore, you ought to choose B.

C. Problems for the Argument from Dominating Expected Value

1. The Many-Gods Objection

Let the “God of the Philosophers” be a God that (i) infinitely rewards those who apportion their theistic beliefs to the evidence and (ii) infinitely punishes those who do not. (By Pascal’s lights, this second group includes theists and atheists; the first group includes only strict agnostics.)

	<u>Christian God (G_1)</u>	<u>God of the Philosophers (G_2)</u>	<u>$\sim G$</u>
believe in a God (B)	∞	$-\infty$	10
suspend belief ($\sim B_1$)	-10,000 (?)	∞	175 (?)
disbelieve ($\sim B_2$)	-10,000	$-\infty$	200

Assuming that $P(G_1) > 0$ and $P(G_2) > 0$, which option maximizes expected value?